Econ 476: Industrial Organization

Monopoly

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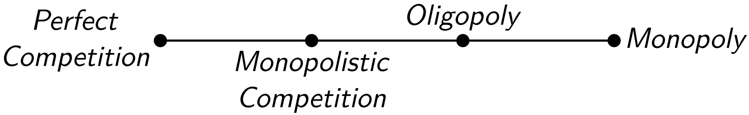
Lecture 02

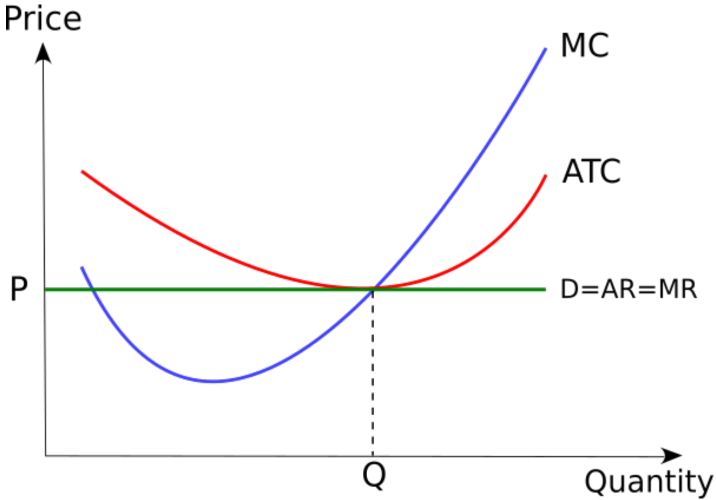
Spectrum


Perfect Competition
Monopolistic Competition
Oligopoly
Monopoly

Perfect competition

- Number of firms/consumers are large
  - no individual firm has market power
- Faces *horizontal* demand curve
- Can only choose *quantity*
- "Invisible hand"

Perfect competition



Example - perfect comp

- Inverse demand: $P = a$
- Cost: $TC(Q) = F + cQ^2$
- Solve for $\pi(Q)$, MR, $MC(Q)$, Q^*, and $\pi(Q^*)$.

Example - perfect competition

Solution

- $\pi(Q) = [aQ] - \left[F + cQ^2\right]$
- $MR = a$
- $MC(Q) = 2cQ$
- $Q^* = \frac{a}{2c}$
- $\pi(Q^*) = \frac{a^2}{4c} - F$

The profit-maximizing output is:

$$Q^* = \begin{cases} \frac{a}{2c} & \text{if } F \leq \frac{a^2}{4c} \\ 0 & \text{otherwise} \end{cases}$$

Monopoly - theory

- Single firm
- Can choose either *price* or *quantity*
- Faces a *downward sloping* demand curve
- The monopoly's profit-maximization problem is either

$$\max \pi(Q) = TR(Q) - TC(Q)$$

$$\max \pi(P) = TR(P) - TC(P)$$

Monopoly - theory

- Two necessary conditions for $Q^M > 0$:
    1. $\pi\left(Q^M\right) > 0$
    2. $0 = \frac{\partial TR\left(Q^M\right)}{\partial Q} - \frac{\partial TC\left(Q^M\right)}{\partial Q} = MR\left(Q^M\right) - MC\left(Q^M\right)$
       $\Rightarrow MR\left(Q^M\right) = MC\left(Q^M\right)$

- [graphs]
- Note: If either of these conditions are not satisfied, then $Q^M = 0$ is the profit-maximizing monopoly output.

Algorithm

How to solve for optimal monopoly profits:

- Step 1: write out profit function $\rightarrow \pi(Q) = TR(Q) - TC(Q)$
- Step 2: derive MR and MC
- Step 3a: equate MR and MC
- Step 3b: solve for Q^M
- Step 4: substitute Q^M into profit function
- Step 5: simplify!

Example - monopoly (quantity)

- Inverse demand: $P(Q) = a - bQ$
- Cost: $TC(Q) = F + cQ^2$
- Solve for $\pi(Q)$, $MR(Q)$, $MC(Q)$, Q^M, P^M, and $\pi\left(Q^M\right)$.

Example - monopoly (quantity)

Solution

- $\pi(Q) = [(a - bQ)\,Q] - \left[F + cQ^2\right]$
- $MR(Q) = a - 2bQ$
- $MC(Q) = 2cQ$
- $Q^M = \frac{a}{2(b+c)}$
- $P^M = \frac{a(b+2c)}{2(b+c)}$
- $\pi\left(Q^M\right) = \frac{a^2}{4(b+c)} - F$

The profit-maximizing output is:

$$Q^M = \begin{cases} \frac{a}{2(b+c)} & \text{if } F \leq \frac{a^2}{4(b+c)} \\ 0 & \text{otherwise} \end{cases}$$

Note: Given the same demand and cost, $Q^M < Q^{PC}$.

Example - monopoly (price)

- Inverse demand: $P(Q) = a - bQ$
- Cost: $TC(Q) = F + cQ^2$
- Solve for $\pi(P)$, $MR(P)$, $MC(P)$, Q^M, P^M, and $\pi\left(P^M\right)$.

Example - monopoly (price)

Solution

- $\pi(P) = \left[P\left(\frac{a-P}{b}\right)\right] - \left[F + c\left(\frac{a-P}{b}\right)^2\right]$
- $MR(P) = \frac{a-2P}{b}$
- $MC(P) = \frac{2c(a-P)}{b^2}$
- $Q^M = \frac{a}{2(b+c)}$
- $P^M = \frac{a(b+2c)}{2(b+c)}$
- $\pi\left(P^M\right) = \frac{a^2}{4(b+c)} - F$

The profit-maximizing output is:

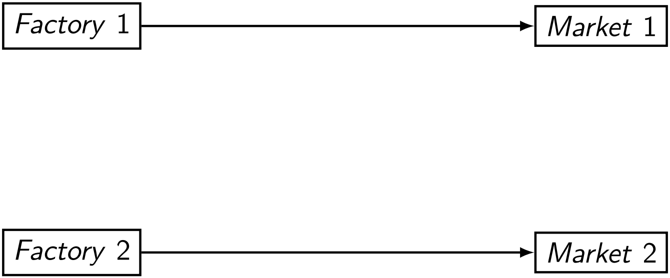
$$Q^M = \begin{cases} \frac{a}{2(b+c)} & if\ F \leq \frac{a^2}{4(b+c)} \\ 0 & otherwise \end{cases}$$

Price discrimination

- Price discrimination is *usually* legal.
- Price discrimination becomes unlawful when the purpose/result is to reduce market competition.
  - Also illegal to price discriminate based on race, religion, nationality, or gender.
- To effectively price discriminate, arbitrage must be impossible (or severely limited).

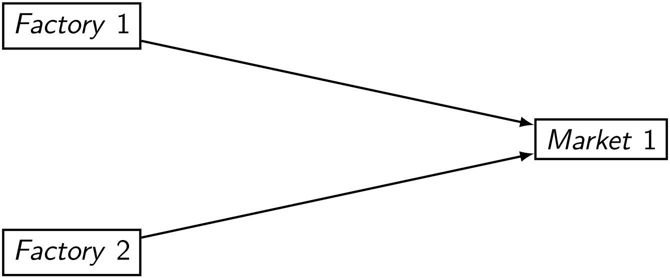
Firm structure (1)

- $MC_1 = MR_1$ and $MC_2 = MR_2$



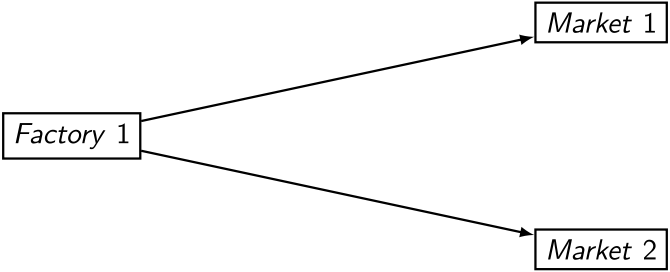
Firm structure (2)

- $MC_1 = MC_2 = MR_1$



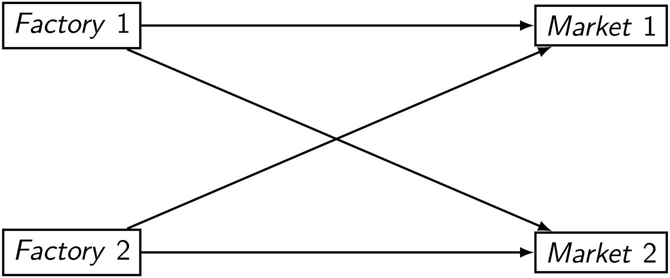
Firm structure (3)

- $MC_1 = MR_1 = MR_2$



Firm structure (4)

- $MC_1 = MC_2 = MR_1 = MR_2$



Updated algorithm

How to solve for optimal monopoly profits (*multiple markets/factories*):

- Step 1: write out profit function $\rightarrow \pi(Q) = TR(Q) - TC(Q)$
- Step 2: derive $MR(q_i)$ and $MC_j(Q)$ $\forall$ i and j
- Step 3a: equate $MR(q_i)$ and $MC_j(Q)$ $\forall$ i and j
- Step 3b: solve for q_i^M $\forall$ i
- Step 4: substitute q_i^M into profit function $\forall$ i
- Step 5: simplify!

Example - 2 markets

- Inverse demand: $p_1(Q) = a - cq_1$ and $p_2(Q) = b - dq_2$
- Cost: $TC(Q) = Q^2$ where $Q = q_1 + q_2$
- [graphs]
- Solve for $\pi(Q)$, $MR(q_i)$, $MC(q_i)$, Q^M, P^M, and $\pi(Q^M)$.

Example - 2 markets

Solution

- $\pi(Q) = (a - cq_1)\,q_1 + (b - dq_2)\,q_2 - (q_1 + q_2)^2$
- $MR(q_1) = a - 2cq_1$; $MR(q_2) = b - 2dq_2$
- $MC(Q) = 2(q_1 + q_2)$
- $q_1^M = \frac{a(d+1)-b}{2(c+dc+d)}$; $q_2^M = \frac{b(c+1)-a}{2(c+dc+d)}$
- $p_1^M = a - c\left[\frac{a(d+1)-b}{2(c+dc+d)}\right]$; $p_2^M = b - d\left[\frac{b(c+1)-a}{2(c+dc+d)}\right]$
- $\pi(Q^M) = \left(a - c\left[\frac{a(d+1)-b}{2(c+dc+d)}\right]\right)\left(\frac{a(d+1)-b}{2(c+dc+d)}\right) + \left(b - d\left[\frac{b(c+1)-a}{2(c+dc+d)}\right]\right)\left(\frac{b(c+1)-a}{2(c+dc+d)}\right) - \left(\frac{ad+bc}{2(c+dc+d)}\right)^2$

Consumer surplus

- [graphs]
- $CS = \int_0^{Q^*} D^{-1}(Q)\,dQ - P^*Q^*$
  - Demand: $D(P)$
    - example: $Q = 240 - 2P$
  - Inverse demand: $D^{-1}(Q)$
    - example: $P = 120 - 0.5Q$
- When demand is linear, CS is calculated by simple geometry: $\frac{1}{2} base * height$

Producer surplus

- $PS = P^* Q^* - \int_0^{Q^*} MC(Q)\, dQ$
- In layman's terms, $PS = TR - TVC$
  - TR : total revenue
  - TVC : total variable costs

Profits

- $\pi = P^* Q^* - AC(Q^*) Q^*$
  - AC : average cost, which is defined as $\frac{TC}{Q}$

Note: While $PS \neq \pi$, it is important to note that $\triangle PS = \triangle \pi$!

Note: We will be focusing more on profits during this class.

Social welfare

- π, PS, CS, and DWL, are key metrics in comparing models.
- Moral codes and politics (and a little bit of economics) decides which of these (i.e π or CS) is preferred in any given circumstance.
- From an economics standpoint, we prefer that total surplus (i.e. $\pi + CS$) is maximized.
- Social welfare:

$$W =: CS + \sum_{i=1}^{N} \pi_i$$

  - Essentially, it all comes back to consumers since they own the firms.

Dead weight loss

- DWL is the difference between the efficient market outcome (i.e. perfect competition), and any other inefficient market outcome (i.e. monopolistic competition, oligopoly, and monopoly)
- $DWL = \left(CS^{PC} + PS^{PC}\right) - \left(CS^{nonPC} + PS^{nonPC}\right)$
- However, in this class, we will define dead weight loss as:

$$\begin{aligned} DWL &= W^{PC}(p) - W^{nonPC}(p) \\ DWL &= \left(CS^{PC} + \pi^{PC}\right) - \left(CS^{nonPC} + \pi^{nonPC}\right) \end{aligned}$$

Example - social welfare

A monopolist faces inverse demand $P = 120 - Q$ and cost function cQ. Find the optimal price and quantity. Graph the equilibrium and show consumer surplus, producer surplus and deadweight loss. Compute CS, PS, and DWL. These will be functions of the cost parameter c. Note that since there are no fixed costs, $PS = \pi$.

Example - social welfare

Solution

- $Q^M = 60 - \frac{c}{2}$
- $P^M = 60 + \frac{c}{2}$
- [graphs]
- $CS = \frac{1}{2}\left(60 - \frac{c}{2}\right)^2$
- $PS = \left(60 - \frac{c}{2}\right)^2$
- $DWL = \frac{1}{2}\left(60 - \frac{c}{2}\right)^2$

The solution just happens to be symmetric, but this is a rare occurrence (statistically speaking).

Elasticity

- Elasticity measures the responsiveness of quantity demanded by a change in its price.
  - Specifically, the percentage change in quantity demanded to a 1% change in price.
- Price elasticity of demand:

$$\eta_p = \frac{\partial Q(p)}{\partial p}\frac{p}{Q}$$

- inelastic $\approx$ steep demand curve
  - changing price doesn't really change quantity demanded
- elastic $\approx$ shallow(horizontal) demand curve
  - changing price significantly changes quantity demanded

Elasticity

Definitions:

1. If $\eta_p < -1$, then elastic.
2. If $-1 < \eta_p < 0$, then inelastic.
3. If $\eta_p = -1$, then unit elastic.

MR and elasticity are related:

$$MR = P\left[1 + \frac{1}{\eta_p}\right]$$

Example - elasticity

A monopolist has demand function $Q(P) = aP^{\varepsilon}$ (with $|\varepsilon| > 1$) and total cost function $TC(Q) = cQ$. Calculate the demand elasticity, η_p, and, in turn, calculate MR in terms of P and ε. Find the firm's optimal price as a function of c and ε.

Example - elasticity

Solution

- $\eta_p = \varepsilon$
- $MR = P\left(\frac{\varepsilon+1}{\varepsilon}\right)$
- $P^M = \frac{\varepsilon c}{\varepsilon+1}$