Econ 476: Industrial Organization

Game Theory - Normal form

J. Bradley Eustice

Brigham Young University

Lecture 03

Intro

"Game theory ... is a collection of tools for predicting outcomes for a group of interacting agents, where an action of a single agent directly affects the payoffs of other participating agents."

- IO perfect application

Intro

Let's play a game!

- "Guess two-thirds of the Average"
- Pick a whole number between 1-100.
- The winner is closest to 2/3 of the average guess.

Intro

2 types of games:

- Normal form
  - agents(players) choose actions *simultaneously*
- Extensive form
  - agents may choose actions in different time periods

2 types of actions: pure or mixed

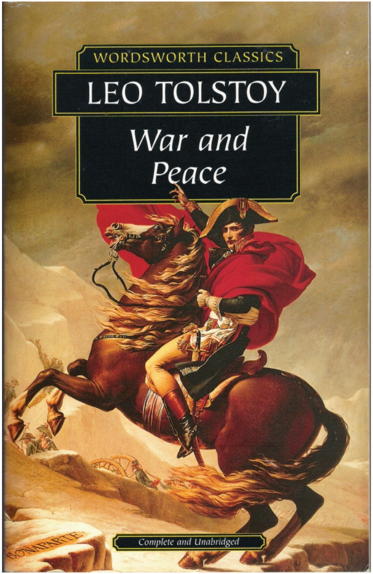
2 types of information: perfect or imperfect

Definition

A normal form game is described by the following:

1. N players whose names are listed in the set $I \equiv \{1, 2, 3, ..., N\}$
2. Each player i, where $i \in I$, has an action set A^i, where $A^i = \{a^i_1, a^i_2, a^i_3, ..., a^i_k\}$
3. List of actions chosen by each player: $a \equiv (a^1, a^2, ..., a^N)$
4. Each player i has a payoff function $\pi^i \in \mathbb{R}$

Example - notation



Example - notation

		RUSSIA			
		WAR		*PEACE*	
FRANCE	*WAR*	1	1	3	0
	PEACE	0	3	2	2

Example - notation

- $N = 2$; $I = \{\ \textsc{France},\ \ \textsc{Russia}\ \}$
- $A^1 = \{\ WAR,\ \ PEACE\ \}$; $A^2 = \{\ WAR,\ \ PEACE\ \}$;
- 4 potential outcomes:
  1. $a = (WAR,\ WAR)$
  2. $a = (WAR,\ PEACE)$
  3. $a = (PEACE,\ WAR)$
  4. $a = (PEACE,\ PEACE)$
- Assume outcome $a = (WAR,\ PEACE)$ is realized.
  - $\pi^1(a) = \pi^1(WAR,\ PEACE) = 3$
  - $\pi^2(a) = \pi^2(WAR,\ PEACE) = 0$

Game?

Is this a game (according to the definition)?

		Nephi			
		BUILD BOAT		*NO BUILD*	
L & L	*BUILD BOAT*	americas	americas	$\Delta\heartsuit$	disfavored
	NO BUILD	shocked	irked	happy	sad

Notation, notation, notation!

- https://www.youtube.com/watch?v=3UO2A2p-19A

- $a^{\neg i} \equiv \left(a^1, ..., a^{i-1}, a^{i+1}, ... a^N\right)$
  - Outcome a can be expressed as $a = \left(a^i, a^{\neg i}\right)$
- Why?
  - When solving these games it is helpful (i.e. necessary) to determine player i's best response to each possible outcome.

Best response functions

- Definition: In an N-player game, the best response function of player i is the function $R^i\left(a^{\neg i}\right)$, that for a given actions $a^{\neg i}$ of players $1,2,...,i-1,i+1,...N$, assigns an action $a^i = R^i\left(a^{\neg i}\right)$ that maximizes player i's payoff $\pi^i\left(a^i, a^{\neg i}\right)$.

Example - best response functions

		Rachel			
		opera		*football*	
Jacob	*opera*	2	1	0	0
	football	0	0	1	2

What are the best response functions for Jacob and Rachel?

- $R^{\text{JACOB}}\left(a^{\text{RACHEL}}\right) = \begin{cases} \textit{opera} & \text{if } a^{\text{RACHEL}} = \textit{opera} \\ \textit{football} & \text{if } a^{\text{RACHEL}} = \textit{football} \end{cases}$
- $R^{\text{RACHEL}}\left(a^{\text{JACOB}}\right) = \begin{cases} \textit{opera} & \text{if } a^{\text{JACOB}} = \textit{opera} \\ \textit{football} & \text{if } a^{\text{JACOB}} = \textit{football} \end{cases}$

Dominant action

- Definition: A particular action $\tilde{a}^i \in A^i$ is said to be a *dominant* action for player i if, regardless of all other player's actions, $\tilde{a}^i$ maximizes player i's payoff, $\pi^i\left(\tilde{a}^i, a^{\neg i}\right)$.
  - Formally, $\pi^i\left(\tilde{a}^i, a^{\neg i}\right) \geq \pi^i\left(a^i, a^{\neg i}\right)$ for every $a^i \in A^i$

Example - dominant action

		Firm Dos	
		low price	*high price*
Firm Uno	*low price*	5, 5	9, 1
	high price	1, 9	7, 7

- Does Firm Uno have a dominant strategy?

Example - dominant action

- Yes!

Solution

- $\pi^{\text{UNO}}(\textit{low price}, \textit{high price}) = 9 > 7 = \pi^{\text{UNO}}(\textit{high price}, \textit{high price})$
- $\pi^{\text{UNO}}(\textit{low price}, \textit{low price}) = 5 > 1 = \pi^{\text{UNO}}(\textit{high price}, \textit{low price})$
- *Low price* is the dominant strategy for both firms (by symmetry)
- *High price* is a strictly dominated strategy
- This is an example of an equilibrium in dominant actions
  - each player plays a dominant action
  - the outcome is $\tilde{a} = \left(\tilde{a}^{\text{UNO}}, \tilde{a}^{\text{DOS}}\right)$

Nash equilibrium

- Definition: An outcome $\hat{a} = (\hat{a}^1, \hat{a}^2, ..., \hat{a}^i, ..., \hat{a}^N)$ (where $\hat{a}^i \in A^i$ for every $i = 1, 2, ..., N$) is said to be a Nash equilibrium (NE) if no player would find it beneficial to deviate provided that all other players do not deviate from their strategies played at the Nash outcome.
  - Formally, $\pi^i\left(\hat{a}^i, \hat{a}^{\neg i}\right) \geq \pi^i\left(a^i, \hat{a}^{\neg i}\right)$ for every $a^i \in A^i$
- Related to dominant action
  - An equilibrium in dominant actions outcome is also a NE. However, a NE outcome is not always an equilibrium in dominant actions.
  - equilibrium in dominant actions $\Rightarrow$ NE, but NE $\nRightarrow$ equilibrium in dominant actions
- Solve for NE using best response functions

Example - NE

		Fancy Car	
		Jack in the Box	*In-n-Out*
College Car	*Jack in the Box*	7 6	3 3
	In-N-Out	2 2	8 10

- Solve for the NE

Example - NE

$$R^{\text{College}}\left(a^{\text{Fancy}}\right)=\begin{cases} In-N-Out & \text{if } a^{\text{Fancy}}=In-N-Out \\ Jack\ in\ the\ Box & \text{if } a^{\text{Fancy}}=Jack\ in\ the\ Box \end{cases}$$

$$R^{\text{Fancy}}\left(a^{\text{College}}\right)=\begin{cases} In-N-Out & \text{if } a^{\text{College}}=In-N-Out \\ Jack\ in\ the\ Box & \text{if } a^{\text{College}}=Jack\ in\ the\ Box \end{cases}$$

- $\pi^{\text{College}}(JintheB, JintheB)=7>2=\pi^{\text{College}}(InNOut, JintheB)$
- $\pi^{\text{College}}(InNOut, InNOut)=8>3=\pi^{\text{College}}(JintheB, InNOut)$
- $\pi^{\text{Fancy}}(JintheB, JintheB)=6>3=\pi^{\text{Fancy}}(InNOut, JintheB)$
- $\pi^{\text{Fancy}}(InNOut, InNOut)=10>2=\pi^{\text{Fancy}}(JintheB, InNOut)$

NE are $\hat{a}=\begin{cases} (Jack\ in\ the\ Box, Jack\ in\ the\ Box) \\ (In-N-Out, In-N-Out) \end{cases}$

Exercise - rock paper scissors

Find all (pure strategy) Nash equilibria.

		Backstreet Boys					
		R		P		S	
	R	0	0	-1	1	1	-1
N'sync	P	1	-1	0	0	-1	1
	S	-1	1	1	-1	0	0

Mixed strategy

- There are no *pure strategy* Nash equilibria in the previous game.
- A mixed strategy assigns a probability to each action in the action set
- John Nash proved that each finite game has *at least* one mixed strategy Nash equilibrium.
  - mixed strategy NE is $\hat{a} = \left(\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right), \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)\right)$
  - any deviation (especially in the long run) would result in lower payoffs

Note: this concept is only introduced for completeness. We will not apply mixed strategies in this course.