Econ 476: Industrial Organization Oligopoly

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Lecture 04

Oligopoly (Simultaneous)

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Cournot

- Number of firms $= [1, \infty)$
 - N = 1 is monopoly
- Homogeneous product
- Firms choose quantity independently and simultaneously
- Each firm has market power
 - changing qⁱ will influence the aggregate price P charged for all total units Q in the market

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Algorithm

How to solve for optimal Cournot profits (*N firms*):

- Step 1: write out profit function for each firm i
 - $\pi_i(q_1, q_2, \cdots, q_N) = TR_i(q_1, q_2, \cdots, q_N) TC_i(q_i)$
- Step 2: take the derivative of π_i with respect to q_i and set to zero ∀ i

 ^{∂π_i}/_{∂α_i} = 0
- Step 3: solve for $q_i^* \forall i$
- Step 4: aggregate q^{*}_i and plug into inverse demand to solve for P^{*}
- ▶ Step 5: substitute in q_i^* and P^* into each firm's profit function $\pi_i \forall i$
- Step 6: simplify!

Note: Price depends on *aggregate* output (it doesn't matter which firm produces the good).

Note:
$$rac{\partial \pi_i}{\partial q_i} = 0$$
 is the same as $MR_i = MC_i$

- Inverse demand: P(Q) = a bQ where $Q = q_1 + q_2$
- Cost: $TC_i(q_i) = c_i q_i$ where i = 1, 2 and $c_1, c_2 \ge 0$
- ► Solve for $\pi_i(q_1, q_2)$, $\frac{\partial \pi_i}{\partial q_i} = 0$, q_i^* , P^* , and $\pi_i^*(q_i^*)$.

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Solution

•
$$\pi_1(q_1, q_2) = [a - b(q_1 + q_2)] q_1 - c_1 q_1;$$

 $\pi_2(q_1, q_2) = [a - b(q_1 + q_2)] q_2 - c_2 q_2$
• $\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c_1 = 0; \ \frac{\partial \pi_2}{\partial q_2} = a - 2bq_1 - bq_2 - c_2 = 0;$
• $q_1^* = \frac{a - 2c_1 + c_2}{3b}; \ q_2^* = \frac{a - 2c_2 + c_1}{3b}$
• $P^* = \frac{a + c_1 + c_2}{3}$
• $\pi_1 = \frac{(a - 2c_1 + c_2)^2}{9b}; \ \pi_2 = \frac{(a - 2c_2 + c_1)^2}{9b}$

Oligopoly (Simultaneous)

A normal form game is described by the following:

- 1. *N* players whose names are listed in the set $I \equiv \{1, 2, 3, ..., N\}$
- 2. Each player *i*, where $i \in I$, has an action set A^i , where $A^i = \{a_1^i, a_2^i, a_3^i, ..., a_k^i\}$
- 3. List of actions chosen by each player: $a \equiv (a^1, a^2, ..., a^N)$
- 4. Each player *i* has a payoff function $\pi^i \in \mathbb{R}$

Does the Cournot equilibrium follow the normal form definition?

Normal form - Cournot

• $N = 2; I = \{ FIRM 1, FIRM 2 \}$

•
$$A^1 = \{1, 2, \cdots, Z_1 < \infty\}; A^2 = \{1, 2, \cdots, Z_2 < \infty\};$$

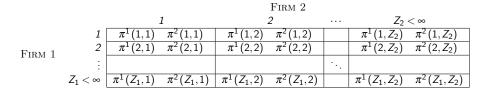
- Infinitely many potential outcomes
- Assume outcome $a = (q_1^*, q_2^*)$ is realized.

•
$$\pi^1(a) = \pi^1(q_1^*, q_2^*) = \frac{(a-2c_1+c_2)^2}{9b}$$

• $\pi^2(a) = \pi^2(q_1^*, q_2^*) = \frac{(a-2c_2+c_1)^2}{9b}$

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Normal form - Cournot



Oligopoly (Simultaneous)

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- There is a better way to solve for NE: use best-response functions!
- Solving for q₁ as a function of q₂ yields the best response function of Firm 1

•
$$R_{F1}(q_2) = q_1 = \frac{a-c_1}{2b} - \frac{1}{2}q_2$$

• Similarly,
$$R_{F2}(q_1) = q_2 = \frac{a-c_2}{2b} - \frac{1}{2}q_1$$

- [graphs]
- A Cournot-Nash equilibrium is characterized by {p^{*}; q₁^{*}, q₂^{*},..., q_N^{*}} where p^{*}, q_i^{*} ≥ 0 ∀i

Bertrand

- Number of firms $= [1, \infty)$
 - N = 1 is monopoly
- Homogeneous product
 - somewhat trivial case
- Firms choose price independently and simultaneously
- 2 assumptions:
 - 1. Consumers purchase from the cheapest seller.
 - 2. If the price is the same among sellers (or a group of sellers), then the market is equally divided among the sellers (group of sellers).

$$\bullet \quad q_i = \begin{cases} 0 & \text{if } p_{\neg i} < p_i \\ \frac{1}{N}Q & \text{if } p_i = p_{\neg i} \\ Q & \text{if } p_i < p_{\neg i} \end{cases} \quad \forall i \text{ where } i \in I = \{1, 2, \dots, N\}$$

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- Economists choose the market structure (monopoly, Cournot, ...) that best approximates the market of interest.
- Different market structures lead to different market outcomes.
 - perfect competition chooses quantity
 - monopoly chooses price or quantity
 - Cournot chooses quantity
 - Bertrand chooses price
- \blacktriangleright Quantity changes may not be feasible in the short run \rightarrow much easier to change prices

- Inverse demand: P(Q) = a bQ where $Q = q_1 + q_2$
- Cost: $TC_i(q_i) = c_i q_i$ where i = 1, 2 and $c_1 = c_2 \ge 0$
- Solve for π_i , p_i^* , q_i^* , and π_i^* .

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Bertrand - 2 firms (same costs)

Solution

•
$$\pi_i = \begin{cases} 0 & \text{if } p_j < p_i \\ p_i \frac{Q}{2} - c_i \frac{Q}{2} & \text{if } p_i = p_j \\ p_i Q - c_i Q & \text{if } p_i < p_j \end{cases}$$

• $p = p_1^* = p_2^* = c_1 = c_2 \equiv c$
• $q_1^* = q_2^* = \frac{a-c}{2b}$
• $\pi_i^* = p_i \left(\frac{a-p_i}{2b}\right) - c_i \left(\frac{a-p_i}{2b}\right) = (p-c) \left(\frac{a-c}{2b}\right) = 0$

Note: With identical costs (for a homogeneous product), the Bertrand outcome is the same as the perfectly competitive market outcome.

Oligopoly (Simultaneous)

Normal form - Bertrand

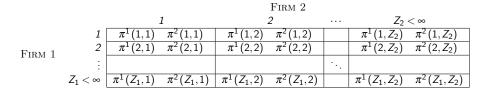
- $N = 2; I = \{ FIRM 1, FIRM 2 \}$
- $A^1 = \{1, 2, \cdots, Z_1 < \infty\}; A^2 = \{1, 2, \cdots, Z_2 < \infty\};$
- Infinitely many potential outcomes
- Assume outcome $a = (p_1^*, p_2^*)$ is realized.

•
$$\pi^1(a) = \pi^1(p_1^*, p_2^*) = 0$$

• $\pi^2(a) = \pi^2(p_1^*, p_2^*) = 0$

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Normal form - Bertrand



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- Again, solve using best response functions
- [graphs]
- ► A Bertrand-Nash equilibrium is characterized by $\{p_1^*, p_2^*, \dots, p_N^*; q_1^*, q_2^*, \dots, q_N^*\}$ where $p_i^*, q_i^* \ge 0 \forall i$

- Inverse demand: P(Q) = a bQ where $Q = q_1 + q_2$
- Cost: $TC_i(q_i) = c_i q_i$ where i = 1, 2 and $c_1 > c_2 \ge 0$
- Solve for π_i , p_i^* , q_i^* , and π_i^*

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Bertrand - 2 firms (diff costs)

Solution

- explore in the homework(!)
- [graphs]

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Oligopoly (Simultaneous)

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- Definitions of π , *CS*, *PS*, and *DWL* are the same as in the monopoly slides
 - π : the profit of each firm
 - ► CS: everything under the demand curve and above the price
 - ► *PS*: everything under the price and above the marginal cost curve
 - DWL: difference between efficient market outcome and any other outcome