

# Econ 476: Industrial Organization

## *Game Theory - Extensive form*

J. Bradley Eustice

Brigham Young University

Lecture 05

2 types of games:

- ▶ Normal form
  - ▶ agents (players) choose actions *simultaneously*
- ▶ Extensive form
  - ▶ agents may choose actions in different time periods

2 types of actions: pure or mixed

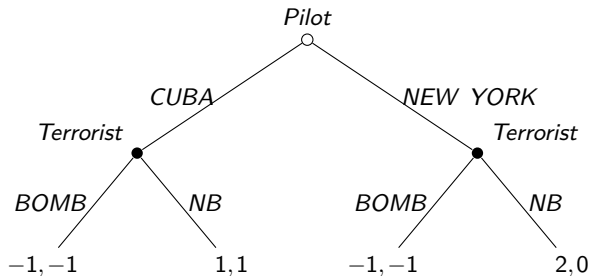
2 types of information: perfect or imperfect

# Definition

An extensive form game is described by the following:

1. A game tree containing a starting node, other decision nodes, terminal nodes, and branches connecting each decision node to successor nodes.
2. A list of  $N \geq 1$  players, indexed by  $i$  where  $i \in I = \{1, 2, \dots, N\}$
3. For each decision node, the name of the player entitled to choose an action.
4. For each decision node, the action set is explicit.
5. The specification of payoffs for each player at each terminal node

# Extensive form



- ▶ Note: Player 1 may not move first

# Strategy

- ▶ Definition: A strategy for player  $i$  is a complete plan of actions, one action for each decision node that the player is entitled to choose an action, regardless of whether or not that decision node is actually reached in the game.
  
- ▶ What are the possible strategies for the terrorist in the previous game?
  - ▶ (B,B)
  - ▶ (B,NB)
  - ▶ (NB,B)
  - ▶ (NB,NB)
  
- ▶ What are the possible strategies for the pilot in the previous game?
  - ▶ NY
  - ▶ CUBA

- ▶ The previous game has eight possible strategic outcomes:
  1. (NY, (B,B))
  2. (NY, (B,NB))
  3. (NY, (NB,B))
  4. (NY, (NB,NB))
  5. (C, (B,B))
  6. (C, (B,NB))
  7. (C, (NB,B))
  8. (C, (NB,NB))

# Normal form representation

		TERRORIST							
		(B,B)	(B,NB)	(NB,B)	(NB,NB)				
PILOT	NY	-1	-1	2	0	-1	-1	2	0
	CUBA	-1	-1	-1	-1	1	1	1	1

- ▶ What are the NE of this game?

# Normal form representation

## Solution

- ▶ There are 3 pure strategy NE:
  1. (NY, (NB,NB))
  2. (NY, (B,NB))
  3. (C, (NB,B))
  
- ▶ Why convert to normal form?
  - ▶ sometimes easier to find NE

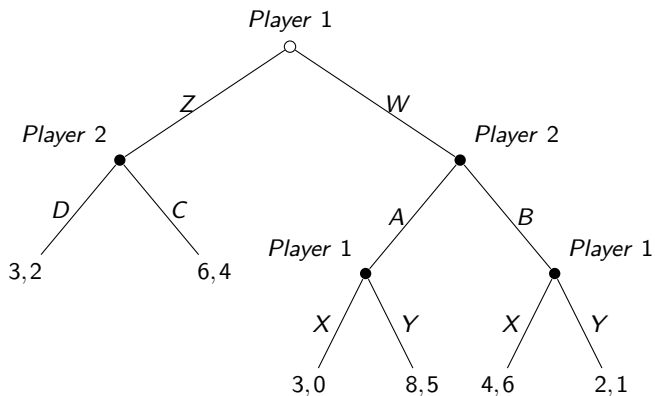


# Subgame

- ▶ Definition: A subgame is a decision node from the original game along with the decision nodes and terminal nodes directly following this node. A subgame is called a proper subgame if it differs from the original game.

# Subgame

How many subgames are in the following game?



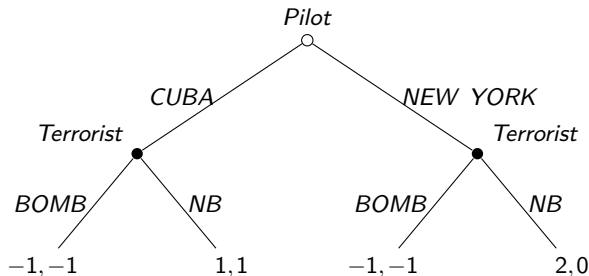
# Subgame perfect equilibrium

- ▶ With multiple NE, it is difficult to predict the outcome of the game (the main purpose of game theory, in general, is to make predictions).
- ▶ Definition: An outcome is said to be a subgame perfect equilibrium (SPE) if it induces a Nash equilibrium in every subgame of the original game

# Threats

- ▶ Credible vs. incredible threats
- ▶ [https://www.youtube.com/watch?v=\\_A2D1PqUGTI](https://www.youtube.com/watch?v=_A2D1PqUGTI)

# Subgame perfect equilibrium



- ▶ Which of the 3 NE are SPE?  
 $(NY, (NB, NB))$ ;  $(NY, (B, NB))$ ;  $(C, (NB, B))$

# Backwards induction

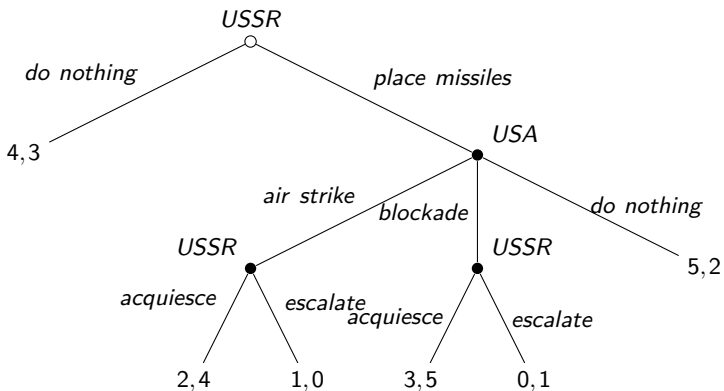
- ▶ In finite games with perfect information, any solution of backwards induction is also a SPE and vice-versa.
  - ▶ SPE  $\Leftrightarrow$  backwards induction solution

## Exercise - backwards induction

During 1962, the Soviet Union installed nuclear missiles in Cuba. When the US found out, President Kennedy discussed the options (i) do nothing, (ii) air strike on the missiles, (iii) a naval blockade of Cuba. JFK decided on the naval blockade. Negotiations ensued, and Khrushchev threatened to escalate the situation; both sides believed that nuclear war was a possibility. Finally, the Soviet Union agreed to remove the missiles if the United States agreed not to invade Cuba. Privately, Kennedy agreed to remove some missiles based in Turkey.

## Exercise - backwards induction

Solve the following game for the SPE using backwards induction:  
(Player 1 is *USSR*, Player 2 is *USA*)





# Repeated games

- ▶ Definition: One-shot games repeated for  $T$  periods, where  $T = [2, \infty)$ .
- ▶ Example: *WAR* and *PEACE* for  $T = \infty$ 
  - ▶ The NE in a one-shot game is  $\{WAR, WAR\}$ .
  - ▶ However, as  $T \rightarrow \infty$ ,  $\{PEACE, PEACE\}$  is an optimal outcome.
- ▶ We will go over repeated games if time permits (Auction and Bargaining section)