Econ 476: Industrial Organization

Differentiated Products

J. Bradley Eustice

Brigham Young University

Lecture 08

Intro

- Most industries produce a large number of similar, but not identical products.
- Only a small subset of *possible* varieties of differentiated products make it to market.

Notation

- $p_1 = \alpha - \beta q_1 - \gamma q_2$ and $p_2 = \alpha - \gamma q_1 - \beta q_2$
- $\beta > 0$ and $\beta^2 > \gamma^2$
  - own-price
  - cross-price
- Measure of differentiation: $\delta = \frac{\gamma^2}{\beta^2}$
  - $\gamma^2 \to 0$, then $\delta \to 0 \Rightarrow$ highly differentiated
  - $\gamma^2 \to \beta^2$, then $\delta \to 1 \Rightarrow$ almost homogeneous

Cournot - 2 firms

- Inverse demand: $p_1 = \alpha - \beta q_1 - \gamma q_2$ and $p_2 = \alpha - \gamma q_1 - \beta q_2$
- Cost: Assume production is costless
- Solve for $\pi_i(q_1, q_2)$, $\frac{\partial \pi_i}{\partial q_i} = 0$, q_i^*, P^* and π_i^*.

Cournot - 2 firms

Solution

- $\pi_1(q_1, q_2) = [\alpha - \beta q_1 - \gamma q_2] q_1$; $\pi_2(q_1, q_2) = [\alpha - \gamma q_1 - \beta q_2] q_2$
- $\frac{\partial \pi_1}{\partial q_1} = \alpha - 2\beta q_1 - \gamma q_2 = 0$; $\frac{\partial \pi_2}{\partial q_2} = \alpha - \gamma q_1 - 2\beta q_2 = 0$
- $q_1^* = \frac{\alpha}{2\beta + \gamma} = q_2^*$
- $p_1^* = \frac{\alpha\beta}{2\beta + \gamma} = p_2^*$
- $\pi_1^* = \frac{\alpha^2 \beta}{(2\beta + \gamma)^2} = \pi_2^*$

Note: The only reason each firm has the same q and π is because $TC_1 = TC_2 = 0$. If they have different costs, then results will be different.

Cournot - 2 firms

- What happens to optimal quantity, price, and profits as γ increases?

Monopoly - 2 products

- How does the outcome change if one firm (a monopoly) supplies the two differentiated products instead of 2 firms competing in a Cournot game?

Monopoly - 2 products

- explore in the homework(!)

Demand function

- Given inverse demand $p_1 = \alpha - \beta q_1 - \gamma q_2$ and $p_2 = \alpha - \gamma q_1 - \beta q_2$, solve for the demand function.

Solution

- $q_1 = a - bp_1 + cp_2$ and $q_2 = a + cp_1 - bp_2$ where
  - $a = \frac{\alpha(\beta-\gamma)}{\beta^2-\gamma^2}$
  - $b = \frac{\beta}{\beta^2-\gamma^2}$
  - $c = \frac{\gamma}{\beta^2-\gamma^2}$
- Note: c does not mean cost

Bertrand - 2 firms

- Inverse demand: $p_1 = \alpha - \beta q_1 - \gamma q_2$ and $p_2 = \alpha - \gamma q_1 - \beta q_2$
- Demand: $q_1 = a - bp_1 + cp_2$ and $q_2 = a + cp_1 - bp_2$
- Cost: Assume production is costless
- Solve for $\pi_i(p_1, p_2)$, $\frac{\partial \pi_i}{\partial p_i} = 0$, p_i^*, q_i^*, and π_i^*.

Bertrand - 2 firms

Solution

- $\pi_1(p_1, p_2) = [a - bp_1 + cp_2]\, p_1$; $\pi_2(p_1, p_2) = [a + cp_1 - bp_2]\, p_2$
- $\frac{\partial \pi_1}{\partial p_1} = a - 2bp_1 + cp_2 = 0$; $\frac{\partial \pi_2}{\partial p_2} = a + cp_1 - 2bp_2 = 0$
- $p_1^* = \frac{a}{2b-c} = p_2^*$
- $q_1^* = \frac{ab}{2b-c} = q_2^*$
- $\pi_1^* = \frac{a^2 b}{(2b-c)^2} = \pi_2^*$
- However, these are not the *final* answers. Need to plug in the values for a, b, and c, and simplify(!).

Note: The only reason each firm has the same p, q, and π is because $TC_1 = TC_2 = 0$. If they have different costs, then results will be different.

Cournot vs. Bertrand

- Recall the best-response functions:
  - Cournot: $q_i = R^i(q_j) = \frac{\alpha - \gamma q_j}{2\beta}$
  - Bertrand: $p_i = R^i(p_j) = \frac{a + c p_j}{2b}$
- [graphs]

Definitions

- Player's strategies are said to be **strategic substitutes** if the best-response functions are downward sloping.
- Player's strategies are said to be **strategic complements** if the best-response functions are upward sloping.

Note: no connection between definitions and whether goods are substitutes/complements

Cournot vs. Bertrand

In a differentiated market:

1. The Cournot market price is higher than the Bertrand market price, $p_{Cournot} > p_{Bertrand}$.
2. The more differentiated the products are, the smaller the difference between the Cournot price and the Bertrand price.
3. The difference in prices between a Cournot and Bertrand game are zero when the products are independent.
4. Profits increase under Cournot and Bertrand games as products become more differentiated.

Bertrand - 2 periods

- Demand: $q_1 = a - bp_1 + cp_2$ and $q_2 = a + cp_1 - bp_2$
- Cost: Assume production is costless
- Let Firm 1 be the leader, and Firm 2 the follower.
- Solve for $\pi_2(p_1, p_2)$, $R^2(p_1)$, $\pi_1(p_1)$, and p_i^*.

Bertrand - 2 periods

Solution

- $\pi_2(p_1, p_2) = [a + cp_1 - bp_2]\, p_2$
- $R^2(p_1) = \frac{a+cp_1}{2b}$
- $\pi_1(p_1) = \left[a - bp_1 + c\left(\frac{a+cp_1}{2b}\right)\right] p_1$
- $p_1^* = \frac{a(c+2b)}{2(2b^2-c^2)}$
- $p_2^* = \frac{a\left(4b^2-c^2+2bc\right)}{4b(2b^2-c^2)}$

Bertrand - 1 vs 2 periods

- Now let $a = 168$, $b = 2$, and $c = 1$.
- $p_1^* = 60$; $p_2^* = 56$
- $\pi_1^* = 6300$; $\pi_2^* = 6498$
- Recall that the one-period solutions were:
  - $p_1^{one-shot} = p_2^{one-shot} = \frac{a}{2b+c} = 56$
  - $\pi_1^{one-shot} = \pi_2^{one-shot} = \frac{a^2 b}{(2b+c)^2} = 6272$

Bertrand - 1 vs 2 periods

In a differentiated market:

1. There is a *second*-mover advantage in the sequential Bertrand game.
2. Both firms collect higher profits in the sequential Bertrand game vs the simultaneous Bertrand game.
3. Compared to the simultaneous Bertrand game, the increase in profit for the leader is smaller than for the follower.