Econ 476: Industrial Organization

Research and Development

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Lecture 11

Intro

- https://www.strategyand.pwc.com/innovation1000

Intro

- Research and development can be classified into two types:
  1. *process innovation*: investment to find cost-reducing technologies for existing products
  2. *product innovation*: investment to find technologies to produce new products

Intro

- Suppose that inverse demand for a given product is $P = a - Q$ and unit production cost of said product is c.
  - A *process innovation* decreases the value of c.
  - A *product innovation* increases the value of a.
    - or it can be argued that it decreases the value of c from infinity to something more manageable

Social welfare

- Assume inverse demand is $p = a - bQ$.
- Let a_0, c_0, and p_0 be pre-innovation values and a_1, c_1, and p_1 be post-innovation values.
- What is the social value of innovation if there is only one firm in the market (monopoly)?
- What is the social value of innovation in a perfectly competitive market?
- [graphs]

Social welfare

Results:

- Monopoly: $\left[\frac{1}{2}(a_1 - p_1)\left(\frac{a_1 - p_1}{b}\right) + (p_1 - c_1)\left(\frac{a_1 - c_1}{b}\right)\right] - \left[\frac{1}{2}(a_0 - p_0)\left(\frac{a_0 - c_0}{b}\right) + (p_0 - c_0)\left(\frac{a_0 - c_0}{b}\right)\right]$
- Perfectly competitive: $\left[\frac{1}{2}(a_1 - c_1)^2\right] - \left[\frac{1}{2}(a_0 - c_0)^2\right]$
- We can generalize the results given the slope of the inverse demand curve (i.e. $b = 1$)

$$\left[\frac{1}{2}(a_1 - p_1)^2 + (a_1 - p_1)(p_1 - c_1)\right] - \left[\frac{1}{2}(a_0 - p_0)^2 + (a_0 - p_0)(p_0 - c_0)\right]$$

Math review

- If events A and B are independent, then $P(A \cap B) = P(A \& B) = P(A)P(B)$.
- Expected value: $E(X) = x_1\omega_1 + x_2\omega_2 + \cdots + x_N\omega_N$, where ω_i is the probability such that $0 < \omega_i < 1$ and $\sum_{i=1}^{N} \omega_i = 1$, and $x_i \in X$.
  - Prove: $1 + 2\omega + 3\omega^2 + \cdots = \sum_{t=1}^{\infty} t\omega^{t-1} = \frac{1}{(1-\omega)^2}$

Discovery

- Assume that α is the probability of discovering a new technology.
- The probability of discovery in period 1 is α.
- The probability of discovery in period 2 is $(1-\alpha)\alpha$.
- What is the probability of discovery in period N?
- Now let's determine the *expected* date (period) of discovery, $E_1(T)$, for a firm investing in R&D.

Discovery

- $E_1(T) = \alpha \sum_{t=1}^{\infty} t(1-\alpha)^{t-1} = \frac{\alpha}{[1-(1-\alpha)]^2} = \frac{1}{\alpha}$
- Now assume that 2 firms are investing in R&D.
  - The probability that neither firm discovers during a particular period is $(1-\alpha)^2$.
  - The probability that at least one firm discovers during a particular period is $\left[1-(1-\alpha)^2\right] = \alpha(2-\alpha)$.
- What is the expected date of discovery, $E_2(T)$?

Discovery

- $E_2(T) = \alpha(2-\alpha)\sum_{t=1}^{\infty} t\left[(1-\alpha)^2\right]^{t-1} = \frac{\alpha(2-\alpha)}{\left[1-(1-\alpha)^2\right]^2} = \frac{1}{\alpha(2-\alpha)}$
- Now assume there are N firms investing in R&D.
- What is the expected date of discovery, $E_N(T)$?
  - solve in the homework(!)

R&D - model

Consider a 2-stage game with 2 firms. In period one, each firm determine how much to invest in a cost-reducing technology. In period 2, the firms compete by choosing quantity on the homogeneous good (Cournot).

- Inverse demand: $P = 100 - Q$
- Production cost: $c_i(x_i, x_j) = 50 - x_i - \beta x_j$
  - x_i is the amount spent on R&D for firm i
  - $0 < \beta < 1$: spillover effects from the other firm's R&D
- R&D cost: $\frac{(x_i)^2}{2}$

- Assume that both firms run their labs independently. Solve for the Nash equilibrium.

Remember: sequential games are solved backwards

R&D - noncooperative

Results:

(Period 2)

- $\pi_i = [100 - q_i - q_j]\, q_i - c_i q_i$
- $q_i^* = \frac{100 + c_j - 2c_i}{3}$; $P^* = \frac{100 + c_1 + c_2}{3}$
- $\pi_i^* = \frac{(100 + c_j - 2c_i)^2}{9}$

R&D - noncooperative

Results:

(Period 1)

- $\pi_i = \frac{1}{9}\left[100 + (50 - x_j - \beta x_i) - 2(50 - x_i - \beta x_j)\right]^2 - \frac{(x_i)^2}{2}$
- $x_i = \frac{2}{9}\left[\frac{50(2-\beta)+(2\beta-1)(2-\beta)x_j}{1-(2-\beta)^2}\right]?$
- $x_1^* = x_2^* = x_{nc} = \frac{50(2-\beta)}{4.5-(2-\beta)(1+\beta)}$

Now let's assume that both firms choose their respective R&D investment levels to maximize joint profits.

R&D - cooperative

Results:
(Period 2)

- same as in the noncooperative case

R&D - cooperative

Results:

(Period 1)

- $\max_{x_1, x_2} (\pi_1 + \pi_2)$
- $x_i = \frac{2}{9}\left[\frac{100(2-\beta)+(\beta+1)(2-\beta)x_j}{1-(2-\beta)^2-(2\beta-1)^2}\right]$?
- $x_1^* = x_2^* = x_c = \frac{50(\beta+1)}{4.5-(\beta+1)^2}$

R&D - non/cooperative

Results:

1. If $\beta = \frac{1}{2}$, then $x_c = x_{nc}$.
2. If $\beta > \frac{1}{2}$ (i.e. spillover effects are large), then $x_c > x_{nc}$.
3. If $\beta < \frac{1}{2}$ (i.e. spillover effects are small), then $x_c < x_{nc}$.
4. Cooperation in R&D increases firms profits.

Prove in the homework(!)

Patents - Intro

U.S. Constitution, Section 8:

> *"The Congress shall have Power ... To promote the Progress of Science and useful Arts, by securing for limited Times to Authors and Inventors the exclusive Right to their respective Writings and Discoveries."*

Patents - definition

- Patents grant temporary monopoly rights to inventors.
  - lasts 17 years in the US
  - [graph]
- The patent system balances 2 major social goals:
  1. provide firms with the incentives for producing know-how
  2. disseminate this know-how to the public as quickly as possible

Patents - criteria

- What can be patented?
  - products
  - processes
  - plants
  - designs
  - BUT, *not* abstract ideas (like proofs or mathematical formulas)
- Must satisfy 3 criteria:
  1. novelty
  2. non obviousness
  3. usefulness

Patents - model

- What is the optimal value for T?
- [build model]

Patents - model

Consider a 2-stage game. In the first stage, the government sets the duration, T, of the patent life. In the second stage, the firm decides how much to invest in R&D. In periods $t = 1, \ldots, T$ the firm is protected by the patent and collects monopoly profits.

- inverse demand: $P = a - Q$
- R&D cost: $\frac{x^2}{2}$
- unit cost: c or $c - x$ with innovation
- [graph]
- discount rate: ρ, where $0 < \rho < 1$

Solve for the Nash equilibrium (i.e x^* and T^*).

Patents

Results:
(Period 2)

- $\pi = \sum_{t=1}^{T} \rho^{t-1} M(x) - \frac{x^2}{2}$
  $= \frac{1-\rho^T}{1-\rho}(a-c)x - \frac{x^2}{2}$
- $R^x = \frac{1-\rho^T}{1-\rho}(a-c)$

What happens to x as $T\uparrow$? $\rho\uparrow$? $a\uparrow$? $c\uparrow$?

Patents

Results:
(Period 1)

- $\max_{T} SW = \sum_{t=1}^{\infty} \rho^{t-1} [CS_0 + M(R^x)] + \sum_{t=T+1}^{\infty} \rho^{t-1} DL(R^x) - \frac{(R^x)^2}{2}$
  $$= \frac{(a-c)^2}{1-\rho} \left[\frac{1}{2} + \frac{1-\rho^T}{1-\rho} - \frac{\left(1-\rho^T\right)^2 \left(\rho^T + \rho - 1\right)}{2(1-\rho)^2} \right]$$
- $T^* = \frac{ln\left(3+\sqrt{6+\rho^2-6\rho}-\rho\right)-ln(3)}{ln(\rho)}$
- $x^* = \frac{1-\rho^{T^*}}{1-\rho}(a-c)$