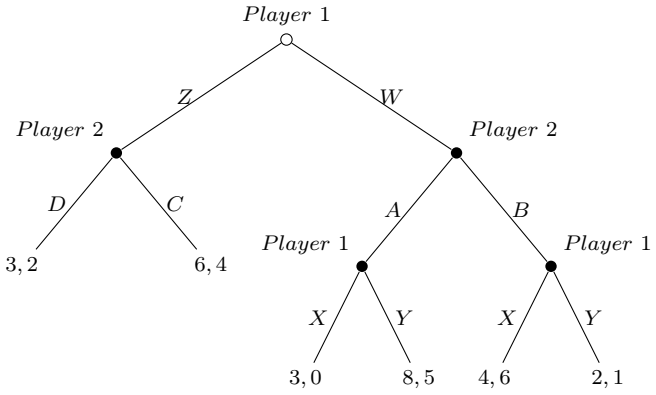
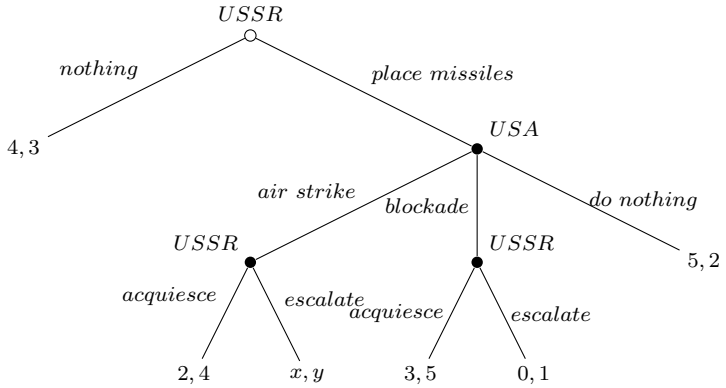
Problem Set 3
Econ 476
Fall 2017
Due: October 19 (Thursday) at 4:00pm, FOB 130

1. Revisit the example we did in class (Extensive form slides: Subgame). What is the subgame perfect Nash equilibrium of the following? What are the payoffs for each player at the SPE?



2. Revisit the example we did in class (Extensive form slides: Exercise - backwards induction). For what values of x and y is $\{(place\ missiles,\ (escalate,\ acquiesce)),\ air\ strike\}$ an SPE? Note that the notation of the SPE is $\{player\,1\,strategy,\ player\,2\,strategy\}$. **Hint**: Use $\leq$ and $\geq$, not $<$ and $>$.



3. Consider a 3-period sequential-moves (Stackelberg-ish) game with 3 firms: Firm A (the leader), Firm B, and Firm C. Firm A moves in the first period, Firm B moves in the second period, and Firm C moves in the last period. At the end of the third period, firms sell their output and collect their profits. Assume that the market inverse demand is given by $P = a - bQ$ and the cost of production for each firm is $c_A q_A$, $c_B q_B$, and $c_C q_C$, respectively. Also assume that each firm competes by choosing quantity. **Hint**: Solve backwards (period 3, then period 2, and lastly solve the first period).

   (a) What is the profit function of Firm C? Solve for the best-response function, $R^C(q_A, q_B)$, of Firm C.

   (b) What is the profit function of Firm B? Solve for the best-response function, $R^B(q_A)$, of Firm B.

   (c) What is the profit function of Firm A?

   (d) Solve for the optimal output level of each firm (i.e. q_A^*, q_B^*, q_C^*) and the market clearing price, P^*.

(e) Now let $a = 120$, $b = 1$, and $c_A = c_B = c_C = 0$. What is the optimal output for each firm and market clearing price?

(f) Solve for the profits of each firm.

(g) Now assume that instead of competing sequentially, the 3 firms compete simultaneously (i.e Cournot competition). Solve for the optimal output level of each firm and market clearing price using the parameter values presented in part (e).

(h) Compare the quantities, price and profits of each firm for the sequential game and the simultaneous game. Is there a first-mover advantage?

4. Consider a monopolist that would like to bring two brand new, but differentiated, products to market, so the only competition will be between the two products. The monopolist has estimated (inverse) demand for each product to be $p_1 = \alpha - \beta q_1 - \gamma q_2$ and $p_2 = \alpha - \gamma q_1 - \beta q_2$. The monopolist is unsure whether offering both products would increase or decrease profits and he/she has come to you for help. Assume production is costless.

   (a) Write out the profit function for the monopolist.

   (b) Solve for the optimal quantities as a function of α, β, and γ

   (c) Solve for the monopoly profits if both products are brought to market.

   (d) Solve for monopoly profits if only one product is brought to market (choose either product 1 or 2 – it doesn't matter as the results will be the same).

   (e) Now let $\alpha = 290$, $\beta = 3$, and $\gamma = 1$. What is the degree of differentiation (i.e. δ from the slides)?

   (f) Should the monopolist bring both products to market?

   (g) Assume that α and β still equal the values in part (e). For what value of γ is the monopolist indifferent between bringing one or both products to market.

5. Suppose that there are only two firms offering puppies for hire during finals week, called (1) See Spot Walk and (2) Dogs Unleashed. Let α_i denote the advertising level of firm i. Assume that the profits of the firms are affected by the advertising levels taken by the firms. Let $\pi_{See\ Spot\ Walk}(\alpha_1, \alpha_2) = 4\alpha_1 + 3\alpha_1\alpha_2 - \alpha_1^2$ and $\pi_{Dogs\ Unleashed}(\alpha_1, \alpha_2) = 2\alpha_2 + \alpha_1\alpha_2 - \alpha_2^2$.

   (a) Calculate the best-response function of each firm. Draw the best-response function in the $(\alpha_1 - \alpha_2)$ space with α_1 on the vertical axis and α_2 on the horizontal axis.

   (b) Are the firms strategies strategic complements or strategic substitutes?

   (c) Find the Nash equilibrium advertising levels. Also calculate the firms' Nash equilibrium profit levels.

6. Consider 2 firms (Firm 1 and Firm 2) each selling a differentiated product and competing on price (Bertrand). Let market demand for each product be $q_1 = a - bp_1 + cp_2$ and $q_2 = a + cp_1 - bp_2$ where $a = \frac{\alpha(\beta-1)}{\beta^2-\gamma^2}$, $b = \frac{\beta}{\beta^2-\gamma^2}$, and $c = \frac{\gamma}{\beta^2-\gamma^2}$. Assume cost of production is $\kappa_i q_i$ for each firm. **Note**: Don't plug in the values for a, b, and c until part (b).

   (a) Set-up the profit function for each firm. Note that neither should be a function of q_i.

   (b) Solve for p_1^* and p_2^*. Simplify!

   (c) Let $\alpha = 500$, $\beta = 3$, and $\gamma = 1$. Also let $\kappa_1 = 1$ and $\kappa_2 = 2$. What are the profits for each firm?