Econ 476: Industrial Organization

Oligopoly

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Lecture 04

Cournot

- Number of firms $= [1, \infty)$
  - $N = 1$ is monopoly
- Homogeneous product
- Firms choose **quantity** *independently* and *simultaneously*
- Each firm has market power
  - changing q^i will influence the aggregate price P charged for all total units Q in the market

Algorithm

How to solve for optimal Cournot profits (*N firms*):

- Step 1: write out profit function for each firm i
  - $\pi_i(q_1, q_2, \cdots, q_N) = TR_i(q_1, q_2, \cdots, q_N) - TC_i(q_i)$
- Step 2: take the derivative of π_i with respect to q_i and set to zero $\forall\ i$
  - $\frac{\partial \pi_i}{\partial q_i} = 0$
- Step 3: solve for $q_i^* \ \forall\ i$
- Step 4: aggregate q_i^* and plug into inverse demand to solve for P^*
- Step 5: substitute in q_i^* and P^* into each firm's profit function $\pi_i \ \forall\ i$
- Step 6: simplify!

Note: Price depends on *aggregate* output (it doesn't matter which firm produces the good).
Note: $\frac{\partial \pi_i}{\partial q_i} = 0$ is the same as $MR_i = MC_i$

Cournot - 2 firms

- Inverse demand: $P(Q) = a - bQ$ where $Q = q_1 + q_2$
- Cost: $TC_i(q_i) = c_i q_i$ where $i = 1, 2$ and $c_1, c_2 \geq 0$
- Solve for $\pi_i(q_1, q_2)$, $\frac{\partial \pi_i}{\partial q_i} = 0$, q_i^*, P^*, and $\pi_i^*(q_i^*)$.

Cournot - 2 firms

Solution

- $\pi_1(q_1, q_2) = [a - b(q_1 + q_2)] q_1 - c_1 q_1;$
  $\pi_2(q_1, q_2) = [a - b(q_1 + q_2)] q_2 - c_2 q_2$
- $\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c_1 = 0;\ \frac{\partial \pi_2}{\partial q_2} = a - 2bq_1 - bq_2 - c_2 = 0;$
- $q_1^* = \frac{a-2c_1+c_2}{3b};\ q_2^* = \frac{a-2c_2+c_1}{3b}$
- $P^* = \frac{a+c_1+c_2}{3}$
- $\pi_1 = \frac{(a-2c_1+c_2)^2}{9b};\ \pi_2 = \frac{(a-2c_2+c_1)^2}{9b}$

Normal form (reference)

A normal form game is described by the following:

1. N players whose names are listed in the set $I \equiv \{1, 2, 3, ..., N\}$
2. Each player i, where $i \in I$, has an action set A^i, where $A^i = \left\{a^i_1, a^i_2, a^i_3, ..., a^i_k\right\}$
3. List of actions chosen by each player: $a \equiv (a^1, a^2, ..., a^N)$
4. Each player i has a payoff function $\pi^i \in \mathbb{R}$

Does the Cournot equilibrium follow the normal form definition?

Normal form - Cournot

- $N = 2$; $I = \{ \text{Firm } 1, \quad \text{Firm } 2 \}$
- $A^1 = \{1, 2, \cdots, Z_1 < \infty\}$; $A^2 = \{1, 2, \cdots, Z_2 < \infty\}$;
- Infinitely many potential outcomes
- Assume outcome $a = (q_1^*, q_2^*)$ is realized.
  - $\pi^1(a) = \pi^1(q_1^*, q_2^*) = \frac{(a-2c_1+c_2)^2}{9b}$
  - $\pi^2(a) = \pi^2(q_1^*, q_2^*) = \frac{(a-2c_2+c_1)^2}{9b}$

Normal form - Cournot

		Firm 2			
		1	*2*	$\cdots$	$Z_2 < \infty$
Firm 1	*1*	$\pi^1(1,1) \quad \pi^2(1,1)$	$\pi^1(1,2) \quad \pi^2(1,2)$		$\pi^1(1,Z_2) \quad \pi^2(1,Z_2)$
	2	$\pi^1(2,1) \quad \pi^2(2,1)$	$\pi^1(2,2) \quad \pi^2(2,2)$		$\pi^1(2,Z_2) \quad \pi^2(2,Z_2)$
	$\vdots$			$\ddots$	
	$Z_1 < \infty$	$\pi^1(Z_1,1) \quad \pi^2(Z_1,1)$	$\pi^1(Z_1,2) \quad \pi^2(Z_1,2)$		$\pi^1(Z_1,Z_2) \quad \pi^2(Z_1,Z_2)$

Normal form - Cournot

- There is a better way to solve for NE: use best-response functions!
- Solving for q_1 as a function of q_2 yields the best response function of Firm 1
  - $R_{F1}(q_2) = q_1 = \frac{a-c_1}{2b} - \frac{1}{2}q_2$
  - Similarly, $R_{F2}(q_1) = q_2 = \frac{a-c_2}{2b} - \frac{1}{2}q_1$
- [graphs]
- A Cournot-Nash equilibrium is characterized by $\left\{p^*; q_1^*, q_2^*, \ldots, q_N^*\right\}$ where $p^*, q_i^* \geq 0 \;\forall\, i$

Bertrand

- Number of firms $= [1, \infty)$
  - $N = 1$ is monopoly
- Homogeneous product
  - somewhat trivial case
- Firms choose **price** *independently* and *simultaneously*
- 2 assumptions:
  1. Consumers purchase from the cheapest seller.
  2. If the price is the same among sellers (or a group of sellers), then the market is equally divided among the sellers (group of sellers).
  - $q_i = \begin{cases} 0 & \text{if } p_{\neg i} < p_i \\ \frac{1}{N}Q & \text{if } p_i = p_{\neg i} \\ Q & \text{if } p_i < p_{\neg i} \end{cases} \quad \forall\, i \text{ where } i \in I = \{1, 2, \ldots, N\}$

Why Bertrand?

- Economists choose the market structure (monopoly, Cournot, ...) that best approximates the market of interest.
- Different market structures lead to different market outcomes.
  - perfect competition chooses quantity
  - monopoly chooses price or quantity
  - Cournot chooses quantity
  - Bertrand chooses price
- Quantity changes may not be feasible in the short run $\rightarrow$ much easier to change prices

Bertrand - 2 firms (same costs)

- Inverse demand: $P(Q) = a - bQ$ where $Q = q_1 + q_2$
- Cost: $TC_i(q_i) = c_i q_i$ where $i = 1, 2$ and $c_1 = c_2 \geq 0$
- Solve for π_i, p_i^*, q_i^*, and π_i^*.

Bertrand - 2 firms (same costs)

Solution

- $\pi_i = \begin{cases} 0 & \text{if } p_j < p_i \\ p_i \frac{Q}{2} - c_i \frac{Q}{2} & \text{if } p_i = p_j \\ p_i Q - c_i Q & \text{if } p_i < p_j \end{cases}$
- $p = p_1^* = p_2^* = c_1 = c_2 \equiv c$
- $q_1^* = q_2^* = \frac{a-c}{2b}$
- $\pi_i^* = p_i \left(\frac{a-p_i}{2b}\right) - c_i \left(\frac{a-p_i}{2b}\right) = (p-c)\left(\frac{a-c}{2b}\right) = 0$

Note: With identical costs (for a homogeneous product), the Bertrand outcome is the same as the perfectly competitive market outcome.

Normal form - Bertrand

- $N = 2$; $I = \{ \text{Firm 1}, \ \text{Firm 2} \}$
- $A^1 = \{1, 2, \cdots, Z_1 < \infty\}$; $A^2 = \{1, 2, \cdots, Z_2 < \infty\}$;
- Infinitely many potential outcomes
- Assume outcome $a = (p_1^*, p_2^*)$ is realized.
  - $\pi^1(a) = \pi^1(p_1^*, p_2^*) = 0$
  - $\pi^2(a) = \pi^2(p_1^*, p_2^*) = 0$

Normal form - Bertrand

		FIRM 2			
		1	*2*	$\cdots$	$Z_2 < \infty$
FIRM 1	*1*	$\pi^1(1,1)$ $\pi^2(1,1)$	$\pi^1(1,2)$ $\pi^2(1,2)$		$\pi^1(1,Z_2)$ $\pi^2(1,Z_2)$
	2	$\pi^1(2,1)$ $\pi^2(2,1)$	$\pi^1(2,2)$ $\pi^2(2,2)$		$\pi^1(2,Z_2)$ $\pi^2(2,Z_2)$
	$\vdots$			$\ddots$	
	$Z_1 < \infty$	$\pi^1(Z_1,1)$ $\pi^2(Z_1,1)$	$\pi^1(Z_1,2)$ $\pi^2(Z_1,2)$		$\pi^1(Z_1,Z_2)$ $\pi^2(Z_1,Z_2)$

Normal form - Bertrand

- Again, solve using best response functions
- [graphs]
- A Bertrand-Nash equilibrium is characterized by $\left\{p_1^*, p_2^*, \ldots, p_N^*; q_1^*, q_2^*, \ldots, q_N^*\right\}$ where $p_i^*, q_i^* \geq 0 \; \forall \, i$

Bertrand - 2 firms (diff costs)

- Inverse demand: $P(Q) = a - bQ$ where $Q = q_1 + q_2$
- Cost: $TC_i(q_i) = c_i q_i$ where $i = 1, 2$ and $c_1 > c_2 \geq 0$
- Solve for π_i, p_i^*, q_i^*, and π_i^*

Bertrand - 2 firms (diff costs)

Solution

- explore in the homework(!)
- [graphs]

Social welfare

- Definitions of π, CS, PS, and DWL are the same as in the monopoly slides
  - π: the profit of each firm
  - CS: everything under the demand curve and above the price
  - PS: everything under the price and above the marginal cost curve
  - DWL: difference between efficient market outcome and any other outcome